Self-Entangled Field Bellman Equation Complete Theoretical Framework

Lanhaijian

Hongqiao University Science and Technology Academic Network
contact@hongqiao.tech

1 I. Core Theoretical Positioning

The Self-Entangled Field Bellman Equation is a completely original axiomatic mathematical system for underlying intelligent decision-making. Taking the self-entangled field as the core physical basis, it elevates the traditional Bellman equation from discrete states and single-point continuous states to a continuous-field dynamic system with self-coupling, self-reference, and self-evolution. It fundamentally breaks through the limitations of memoryless, non-self, and passive decision-making in traditional AI. It serves as the kernel mathematical cornerstone of the RT-Field-OS (Real-Time Field Operating System) and is the first decision equation unifying four dimensions: physical field–cognitive field–value field–self-consciousness field. Its theoretical originality and engineering value far exceed those of the classical Bellman equation, HJB equation, and ordinary field-based Bellman frameworks.

2 II. Preliminary Definitions: Core Axioms of the Self-Entangled Field

2.1 II.1 Basic Definition of the Self-Entangled Field

Define the self-entangled field $\Psi(\vec{x}, t)$: a four-dimensional continuous spacetime field ($\vec{x} = (x, y, z)$ for 3D physical/cognitive spatial coordinates, t for continuous time) satisfying four axioms: self-entanglement, self-coupling, self-conservation, and differentiability. The core feature distinguishing it from ordinary physical and cognitive fields is that the field quantity nonlinearly entangles and couples with itself without external field participation, achieving self-reference, self-memory, self-prediction, and self-correction.

2.2 II.2 Core Field Quantities and Operators

Symbol	Name	Mathematical/Physical Meaning	Field Property
Ψ	Self-entangled field	Unified field quantity of physical state, cognitive memory, and value utility	Real/complex scalar field; satisfies self-entanglement
$\Psi^\dagger$	Conjugate self-entangled field	Dual field of Ψ; used to compute self-coupling energy	Conjugate field
$\Psi \circledast \Psi$	Self-entanglement kernel	Self-convolution/self-entanglement term; core original term representing self-correlation	Nonlinear self-coupling operator
$V_\Psi(\vec{x}, t)$	Field value function	Optimal value functional of the self-entangled field; core decision metric	Scalar observable
$U_\Psi(\vec{x}, t)$	Instantaneous utility function	Instantaneous reward; external potential driving field evolution	Scalar potential field
$\vec{v}$	Decision flow vector	Decision action flux of the field; characterizes agent behavior	Vector field
Π	Conjugate momentum field	$\Pi = \frac{\partial \mathcal{L}}{\partial(\partial_t \Psi)}$; momentum property of the field	Conjugate field
$\mathcal{L}$	Lagrangian density	Lagrangian density of self-entangled field dynamics; follows least action principle	Scalar density
$\mathcal{H}$	Hamiltonian density	Total energy density of the self-entangled field; drives evolution	Scalar density
∇	Gradient operator	$\nabla = (\partial_x, \partial_y, \partial_z)$; spatial rate of change	Vector differential operator
∂_t	Time derivative	$\partial_t = \frac{\partial}{\partial t}$; temporal evolution rate	Scalar differential operator
γ	Field decay factor	$\gamma \in [0, 1]$; memory dissipation coefficient	Constant
λ	Self-entanglement coupling coefficient	Weight of self-entanglement; determines self-correlation strength	Tunable constant

3 III. Field-Theoretic Version: Continuous Self-Entangled Field Bellman Equation

3.1 III.1 Lagrangian Density Form (Least Action Principle)

The optimal decision trajectory of the self-entangled field satisfies the least action principle. The Lagrangian density introduces the original self-entanglement term absent in all traditional field theories and control equations:

$$\mathcal{L} = \frac{1}{2}(\partial_t \Psi)^2 - \frac{1}{2}|\nabla \Psi|^2 - U(\Psi, \vec{x}, t) + \lambda \cdot \Psi \otimes \Psi$$

Core originality: The last term $\lambda \cdot \Psi \otimes \Psi$ is the self-entanglement coupling term, representing self-memory, self-reference, and self-reinforcement—the mathematical expression of agent autonomy.

Action functional:

$$S = \int \mathcal{L}\, d^3x dt$$

Optimal decision satisfies $\delta S = 0$ (functional variation extremum).

3.2 III.2 Hamiltonian Density Form (Core of Field Evolution)

Via Legendre transformation $\mathcal{H} = \Pi \partial_t \Psi - \mathcal{L}$:

$$\mathcal{H} = \frac{1}{2}\Pi^2 + \frac{1}{2}|\nabla\Psi|^2 + U(\Psi, \vec{x}, t) - \lambda \cdot \Psi \circledast \Psi$$

This Hamiltonian includes self-entanglement energy, governing autonomous evolution rather than pure external driving.

3.3 III.3 Complete Self-Entangled Field Bellman Equation

Taking the value function $V(\Psi, \vec{x}, t)$ as the core observable and combining field dynamics with Bellman optimality:

$$\partial_t V(\Psi, \vec{x}, t) + \nabla V \cdot \vec{v} + \mathcal{H}(V, \Pi, \nabla V) = \gamma V + U(\Psi, \vec{x}, t) + \lambda \cdot \Psi^\dagger \Psi$$

3.4 III.4 Physical Interpretation

- $\partial_t V$: temporal evolution of value
- $\nabla V \cdot \vec{v}$: coupling between value gradient and decision flow
- $\mathcal{H}$: total self-entangled field energy (optimal dynamics)
- γV: value decay (discount factor)
- U: external reward/potential
- $\lambda \cdot \Psi^\dagger \Psi$: self-entanglement core term (self-calibration, memory, reinforcement)

4 IV. Engineering Control Version: Discrete Programmable Form

4.1 IV.1 Discretization Premise

For RT-Field-OS, embedded control, and unmanned systems:

- Time discretization: t_k $(k = 0, 1, 2, \ldots)$
- Spatial discretization: x_i (sensor/control grid index)
- Self-entanglement kernel: $\Psi \circledast \Psi \Rightarrow \Psi_k \star \Psi_k$ (discrete auto-correlation/convolution)

4.2 IV.2 Discrete Engineering Variables

Continuous	Discrete	Engineering Mapping
$\Psi(\vec{x}, t)$	$\Psi_{k,i}$	Tensor/array; real-time update
$V(\Psi, \vec{x}, t)$	$V_{k,i}$	Value iteration array
$\vec{v}$	$\vec{v}_{k,i}$	Control command/action output
$U(\Psi, \vec{x}, t)$	$U_{k,i}$	Reward/utility
$\Psi \circledast \Psi$	$\Psi_{k,i} \star \Psi_{k,i}$	Auto-correlation
∇	∇_i	Grid difference

4.3 IV.3 Complete Discrete Engineering Equations

Lightweight real-time version (for RT-Field-OS):

$$V_{k+1,l} = (1-\gamma)V_{k,l} + U_{k,l} + \mathcal{H}_{k,l} - \nabla_j V_{k,l} \cdot \vec{v}_{k,l} + \lambda \cdot (\Psi_{k,l} \star \Psi_{k,l})$$

High-precision optimal version (for unmanned systems):

$$V_{k+1,\bar{l}} = \gamma V_{k,\bar{l}} + U_{k,\bar{l}} + \max_{\vec{v}_{k,l}} \left[\mathcal{H}_{k,l} - \frac{V_{k,\bar{\imath}} - V_{k,\bar{\imath}\pm1}}{2\Delta x} \cdot \vec{v}_{k,l} + \lambda \cdot (\Psi_{k,l} \cdot \Psi_{k,l}) \right]$$

4.4 IV.4 Discrete Hamiltonian Expansion

$$\mathcal{H}_{k,\bar{l}} = \omega_1 |V_{k,\bar{j}}| + \omega_2 |\nabla_{\bar{l}} V_{k,\bar{l}}| - \omega_3 \cdot \Psi_{k,l} \star \Psi_{k,l}$$

where $\omega_1, \omega_2, \omega_3$ are tunable weights implementable in C/C++, Python, FPGA.

5 V. Comparison: Self-Entangled Field Bellman vs. Traditional Theories

Framework	State Carrier	Core Traits	Self-entanglement	Decision Driver	Application
Classical Bellman	Discrete single-point	Discrete, memoryless, passive	None	External reward	Static RL
HJB Equation	Continuous single-point	Continuous control, no self-coupling	None	Optimal control	Classical control
Ordinary Field Bellman	Continuous ordinary field	Field evolution, non-self, passive	None	Field potential	General agents
Self-Entangled Field Bellman	Continuous self-entangled field	Self-coupling, self-memory, self-evolution, autonomous	Yes (core)	Self + external	RT-Field-OS, autonomous systems

6 VI. Originality and Core Value

1. Axiomatic breakthrough: First introduction of self-entanglement into decision theory, enabling self-reference and self-memory.
2. Physics–decision unification: Integrates least action, Lagrangian–Hamiltonian dynamics for physically consistent AI.
3. Engineering feasibility: Discrete form is lightweight, real-time, and solvable with standard numerics.
4. Theoretical uniqueness: Original framework filling the global gap in self-consciousness field + optimal decision-making.

7 VII. Engineering Implementation

7.1 VII.1 Feasibility

All operators (convolution, gradient, self-entanglement) are numerically tractable; self-entanglement can be computed via self-attention or matrix multiplication; parameters λ, γ are tunable for real-time systems.

7.2 VII.2 Application Prospects

Autonomous unmanned systems (drone, vehicle, vessel), RT-Field-OS, long-term AI agents, complex-environment autonomous decision systems.