Self-Entangled Field Bellman Equation: Complete Theoretical Framework

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Abstract

Self-Entangled Field Bellman Equation is an original fundamental mathematical axiom system for intelligent decision-making. Based on the self-entangled field, it upgrades the traditional Bellman equation to a self-coupling, self-referencing, self-evolving continuous field dynamic system. It is the core mathematical foundation of RT-Field-OS and the first decision equation that unifies physical field, cognitive field, value field, and self-consciousness field.

1 Core Theoretical Position

Self-Entangled Field Bellman Equation breaks through the limitations of traditional artificial intelligence: no memory, no self-structure, passive decision-making. It realizes the unity of physics and decision intelligence.

2 Basic Definitions

2.1 Self-Entangled Field

The self-entangled field is defined as a four-dimensional continuous space-time field:

$$\Psi(\vec{x}, t), \quad \vec{x} = (x, y, z)$$

It satisfies self-entanglement, self-coupling, self-conservation, and differentiability. The field entangles with itself nonlinearly without external fields.

2.2 Core Symbols

Symbol	Name	Meaning
Ψ	Self-entangled field	Unified state-memory-value field
$\Psi^\dagger$	Conjugate field	Dual field
$\Psi \circledast \Psi$	Self-entangled kernel	Self-coupling operator
V	Field value function	Optimal decision value
U	Utility function	Immediate reward
$\vec{v}$	Decision flow	Action vector
Π	Conjugate momentum	$\Pi = \partial\mathcal{L}/\partial(\partial_t\Psi)$
$\mathcal{L}$	Lagrangian density	Least-action dynamics
$\mathcal{H}$	Hamiltonian density	Total energy density
γ	Discount factor	Memory dissipation
λ	Coupling coefficient	Self-entanglement strength

3 Continuous Field Theory Form

3.1 Lagrangian Density

$$\mathcal{L} = \frac{1}{2}\left(\partial_t\Psi\right)^2 - \frac{1}{2}|\nabla\Psi|^2 - U(\Psi, \vec{x}, t) + \lambda \cdot \Psi \circledast \Psi$$

Action functional:

$$S = \int \mathcal{L}\, d^3x dt, \quad \delta S = 0$$

3.2 Hamiltonian Density

$$\mathcal{H} = \frac{1}{2}\Pi^2 + \frac{1}{2}|\nabla\Psi|^2 + U(\Psi, \vec{x}, t) - \lambda \cdot \Psi \circledast \Psi$$

3.3 Full Self-Entangled Field Bellman Equation

$$\partial_t V + \nabla V \cdot \vec{v} + \mathcal{H} = \gamma V + U + \lambda \Psi^\dagger \Psi$$

4 Discrete Engineering Form

Lightweight real-time version:

$$V_{k+1,l} = (1-\gamma)V_{k,l} + U_{k,l} + \mathcal{H}_{k,l} - \nabla_j V_{k,l} \cdot \vec{v}_{k,l} + \lambda(\Psi_{k,l} * \Psi_{k,l})$$

High-precision version:

$$V_{k+1,\bar{l}} = \gamma V_{k,\bar{l}} + U_{k,\bar{l}} + \max_{\vec{v}} \left[\mathcal{H} - \frac{V_{k,i} - V_{k,i\pm1}}{2\Delta x}\vec{v} + \lambda \Psi_{k,l} \Psi_{k,l} \right]$$

5 Comparison with Traditional Models

Model	State	Self-term	Feature
Classical Bellman	Discrete state	No	Passive
HJB Equation	Continuous point	No	No self-memory
Ordinary Field Bellman	Normal field	No	Passive field
Self-Entangled Bellman	Self-entangled field	Yes	Autonomous, self-memory

6 Originality and Value

1. First self-entanglement decision equation.
2. Unifies physics and reinforcement learning.
3. Directly programmable for engineering.
4. Solves long-term AI forgetting and delay.

7 Applications

Unmanned systems, RT-Field-OS, long-term autonomous agents, complex decision systems.